Derivate delle funzioni elementari

Funzione costante

$$f(x) = c$$

$$\frac{\Delta f}{\Delta x} = \frac{f(x+\Delta x) - f(x)}{\Delta x} = \frac{c-c}{\Delta x} = 0$$

$$\Rightarrow f'(x) = 0$$

Retta

$$f(x) = mx + p$$

$$\frac{\Delta f}{\Delta x} = \frac{f(x+\Delta x) - f(x)}{\Delta x} = \frac{(m(x+\Delta x)+p) - mx - p}{\Delta x} = m$$

$$\Rightarrow f'(x) = m$$

Parabola

$$f(x) = ax^2$$

$$\frac{\Delta f}{\Delta x} = \frac{f(x+\Delta x) - f(x)}{\Delta x} = \frac{a(x+\Delta x)^2 - ax^2}{\Delta x} =$$

$$= \frac{ax^2 + a(\Delta x)^2 + 2ax \cdot \Delta x - ax^2}{\Delta x} = a\Delta x + 2ax$$

$$\Rightarrow \lim_{\Delta x \to 0} \frac{\Delta f}{\Delta x} = 2ax$$

$$\Rightarrow \boxed{D\left(ax^2\right) = 2ax}$$

Cubica

$$f(x) = x^3$$

$$\frac{\Delta f}{\Delta x} = \frac{f(x+\Delta x) - f(x)}{\Delta x} = \frac{(x+\Delta x)^3 - x^3}{\Delta x} =$$

$$= \frac{x^3 + 3x^2 \cdot (\Delta x) + 3x \cdot (\Delta x)^2 + (\Delta x)^3 - x^3}{\Delta x} =$$

$$= 3x^2 + 3x \cdot \Delta x + (\Delta x)^2$$

$$\Rightarrow \lim_{\Delta x \to 0} \frac{\Delta f}{\Delta x} = 3x^2$$

$$\Rightarrow \boxed{D\left(x^3\right) = 3x^2}$$

$$f(x) = x^n \quad n \in \mathbb{N}$$

- Generalizzando si ottiene:

$$D\left(x^4\right) = 4x^3$$

$$D\left(x^5\right) = 5x^4$$

$$\vdots$$

$$\boxed{D\left(x^n\right) = n \cdot x^{n-1}}$$

$$\rightarrow \text{ estensione al caso } n \in \mathbb{Q}$$

Funzioni razionali fratte

$$D\left(\frac{1}{x}\right) = D\left(x^{-1}\right) = -x^{-2} = -\frac{1}{x^2}$$

$$D\left(\frac{1}{x^2}\right) = D\left(x^{-2}\right) = -2x^{-3} = -\frac{2}{x^3}$$

$$\vdots$$

$$D\left(\frac{1}{x^n}\right) = D\left(x^{-n}\right) = -n \cdot x^{-n-1} = -\frac{n}{x^{n+1}}$$

Funzioni irrazionali (radicali)

$$D\left(\sqrt{x}\right) = D\left(x^{1/2}\right) = \frac{1}{2}x^{-1/2} = \frac{1}{2\cdot\sqrt{x}}$$

$$D\left(\sqrt[3]{x}\right) = D\left(x^{1/3}\right) = \frac{1}{3}x^{-2/3} = \frac{1}{3\cdot\sqrt[3]{x^2}}$$

Funzione esponenziale

$$f(x) = e^x$$

$$\frac{\Delta f}{\Delta x} = \frac{f(x+\Delta x) - f(x)}{\Delta x} = \frac{e^{x+\Delta x} - e^x}{\Delta x} = \frac{e^x \cdot \left(e^{\Delta x} - 1\right)}{\Delta x}$$

$$\lim_{\Delta x \to 0} \frac{\Delta f}{\Delta x} = \lim_{\Delta x \to 0} e^x \cdot \frac{\left(e^{\Delta x} - 1\right)}{\Delta x} = e^x$$

essendo (limite notevole)

$$\lim_{x \to 0} \frac{e^x - 1}{x} = 1$$

Funzione logaritmica

$$f(x) = \log x$$

$$\frac{\Delta f}{\Delta x} = \frac{f(x+\Delta x) - f(x)}{\Delta x} = \frac{\log(x+\Delta x) - \log x}{\Delta x} =$$

utilizzando il limite notevole:

$$\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x = e$$

si ottiene:

$$\lim_{\Delta x\to 0}\frac{\Delta f}{\Delta x} = \frac{1}{x} \Rightarrow \boxed{D(\log x) = \frac{1}{x}}$$