Esercizi sulle Derivate

Somma - Differenza

$$y = \log x + \sqrt{x} - x^4$$

$$Dy = D(\log x) + D\left(\sqrt{x}\right) - D\left(x^4\right) =$$

$$= \frac{1}{x} + \frac{1}{2\sqrt{x}} - 4x^3$$

Prodotto

$$y = \sqrt{x} \cdot \log x$$

$$Dy = D\left(\sqrt{x}\right) \cdot \log x + \sqrt{x} \cdot D\left(\log x\right) =$$

$$= \frac{1}{2\sqrt{x}} \cdot \log x + \sqrt{x} \cdot \frac{1}{x}$$

Prodotto

- Conseguenze:

$$D\left(c \cdot f(x)\right) = c \cdot Df(x)$$

$$D\left(4\log x\right) = 4 \cdot \frac{1}{x} = \frac{4}{x}$$

$$D\left(6x^3\right) = 6 \cdot 3x^2 = 18x^2$$

$$D\left(x^3 - 5x^2 + 6x + 4\right) = 3x^2 - 10x + 6$$

Quoziente

$$y = \frac{x^3}{1+x^2}$$

$$Dy = \frac{D\left(x^3\right)\cdot\left(1+x^2\right) - x^3 \cdot D\left(1+x^2\right)}{\left(1+x^2\right)^2} =$$

$$= \frac{3x^2 \cdot \left(1+x^2\right) - x^3 \cdot 2x}{\left(1+x^2\right)^2} = \frac{x^4+3x^2}{\left(1+x^2\right)^2}$$

$$y = \frac{x^2}{\log x}$$

$$Dy = \frac{2x \cdot \log x - x^2 \cdot \frac{1}{x}}{(\log x)^2} = \frac{2x \cdot \log x - x}{(\log x)^2}$$

composizione

$$y = [f(x)]^n$$

$$x \xrightarrow{f} f(x) \xrightarrow{n} [f(x)]^n$$

$$Dy = n \cdot [f(x)]^{n-1} \cdot Df(x)$$

$$D\left[\log^3 x\right] = 3\log^2 x \cdot \frac{1}{x} = \frac{3\log^2 x}{x}$$

$$D\left[\left(1+x^4\right)^3\right] = 3\left(1+x^4\right)^2 \cdot 4x^3$$

composizione

$$y = \log\left[f(x)\right]$$

$$x \xrightarrow{\ f\ } f(x) \xrightarrow{\ \log\ } \log\left[f(x)\right]$$

$$Dy = \frac{1}{f(x)} \cdot Df(x)$$

$$D\left[\log x^3\right] = \frac{1}{x^3} \cdot 3x^2 = \frac{3}{x}$$

$$D\left[\log\left(1+\sqrt{x}\right)\right] = \frac{1}{1+\sqrt{x}} \cdot \frac{1}{2\sqrt{x}}$$

composizione

$$y = \sqrt{f(x)}$$

$$x \xrightarrow{f} f(x) \xrightarrow{\sqrt{\ }} \sqrt{f(x)}$$

$$Dy = \frac{1}{2\sqrt{f(x)}} \cdot Df(x)$$

$$D\left[\sqrt{x^2+x+1}\right] = \frac{1}{2\sqrt{x^2+x+1}} \cdot (2x+1)$$

$$D\left[\sqrt{2^x+x^4}\right] = \frac{1}{2\sqrt{2^x+x^4}} \cdot \left(2^x \cdot \log 2 + 4x^3\right)$$

composizione

$$y = e^{f(x)}$$

$$x \xrightarrow{f} f(x) \xrightarrow{e^{\cdot}} e^{f(x)}$$

$$Dy = e^{f(x)} \cdot Df(x)$$

$$D\left[e^{x^3}\right] = e^{x^3} \cdot 3x^2$$

$$D\left[e^{\sqrt{x}}\right] = e^{\sqrt{x}} \cdot \frac{1}{2\sqrt{x}}$$

$$Da^{f(x)} = a^{f(x)} \cdot \log a \cdot Df(x)$$