

I LIMITI NOTEVOLI

$$\lim_{x \rightarrow 0} \frac{\text{sen}x}{x} = 1 \quad ; \quad \lim_{x \rightarrow 0} \frac{1 - \text{cos}x}{x} = 0 \quad ; \quad \lim_{x \rightarrow 0} \frac{1 - \text{cos}x}{x^2} = \frac{1}{2}$$

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$$\lim_{x \rightarrow 0} \frac{x^2 + x}{2x + \text{sen}x} = \frac{0}{0} \quad \mathbf{F. I.}$$

$$\frac{x^2 + x}{2x + \text{sen}x} = \frac{x(x+1)}{2x + \text{sen}x} = \frac{\frac{x(x+1)}{x}}{\frac{2x + \text{sen}x}{x}} = \frac{x+1}{2 + \frac{\text{sen}x}{x}} \rightarrow$$

$$\lim_{x \rightarrow 0} \frac{x^2 + x}{2x + \text{sen}x} = \lim_{x \rightarrow 0} \frac{x+1}{2 + \frac{\text{sen}x}{x}} = \frac{1}{2+1} = \frac{1}{3}$$

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$$\lim_{x \rightarrow 0} \frac{1 - \text{cos}x - \text{sen}x}{x} = \frac{1 - 1 - 0}{0} = \frac{0}{0} \quad \mathbf{F. I.}$$

$$\lim_{x \rightarrow 0} \left(\frac{1 - \text{cos}x}{x} - \frac{\text{sen}x}{x} \right) = 0 - 1 = -1$$

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$$\lim_{x \rightarrow 0} \frac{\text{sen}2x + x}{x + \text{sen}x} = \frac{0 + 0}{0 + 0} = \frac{0}{0} \quad \mathbf{F. I.}$$

$$\frac{\text{sen}2x + x}{x + \text{sen}x} = \frac{\frac{\text{sen}2x + x}{x}}{\frac{x + \text{sen}x}{x}} = \frac{\frac{\text{sen}2x}{x} + 1}{1 + \frac{\text{sen}x}{x}} = \frac{\frac{2\text{sen}2x}{2x} + 1}{1 + \frac{\text{sen}x}{x}} \rightarrow$$

$$\lim_{x \rightarrow 0} \frac{\text{sen}2x + x}{x + \text{sen}x} = \lim_{x \rightarrow 0} \frac{\frac{2\text{sen}2x}{2x} + 1}{1 + \frac{\text{sen}x}{x}} = \frac{2 \cdot 1 + 1}{1 + 1} = \frac{3}{2}$$

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$$\lim_{x \rightarrow \infty} \left(1 + \frac{1}{x}\right)^x = e \quad ; \quad \lim_{x \rightarrow 0} \frac{\ln(1+x)}{x} = 1 \quad ; \quad \lim_{x \rightarrow 0} \frac{e^x - 1}{x} = 1$$

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$$\lim_{x \rightarrow -\infty} \left(\frac{x-7}{x}\right)^x = 1^\infty \quad \mathbf{F. I.}$$

$\left(\frac{x-7}{x}\right)^x = \left(1 - \frac{7}{x}\right)^x$, ponendo $y = -\frac{x}{7}$, $y \rightarrow +\infty$ per $x \rightarrow -\infty$ e si ha:

$$\lim_{x \rightarrow -\infty} \left(\frac{x-7}{x}\right)^x = \lim_{y \rightarrow +\infty} \left(1 + \frac{1}{y}\right)^{-7y} = \lim_{y \rightarrow +\infty} \left(\left(1 + \frac{1}{y}\right)^y\right)^{-7} = e^{-7}$$

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$$\lim_{x \rightarrow +\infty} \{x[\ln(x+1) - \ln x]\} = +\infty[\infty - \infty] \quad \mathbf{F. I.}$$

$$\lim_{x \rightarrow +\infty} \{x[\ln(x+1) - \ln x]\} = \lim_{x \rightarrow +\infty} \{x[\ln\left(\frac{x+1}{x}\right)]\} = \lim_{x \rightarrow +\infty} \ln\left(1 + \frac{1}{x}\right)^x = \ln(e) = 1$$

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$$\lim_{x \rightarrow +\infty} \left(1 - \frac{2}{x}\right)^x = 1^\infty \quad \mathbf{F. I.}$$

ponendo $t = -\frac{x}{2}$, per $x \rightarrow +\infty$, $t \rightarrow -\infty$ e

$$\lim_{x \rightarrow +\infty} \left(1 - \frac{2}{x}\right)^x = \lim_{t \rightarrow -\infty} \left(1 + \frac{1}{t}\right)^{-2t} = \lim_{t \rightarrow -\infty} \left(1 + \frac{1}{t}\right)^{-2t} = \lim_{t \rightarrow -\infty} \left(\left(1 + \frac{1}{t}\right)^t\right)^{-2} = e^{-2}$$

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$$\lim_{x \rightarrow 0} \frac{\ln(1-4x)}{x} = \frac{0}{0} \quad \mathbf{F. I.}$$

ponendo $t = -4x$, $t \rightarrow 0$ per $x \rightarrow 0$ e il limite diventa:

$$\lim_{x \rightarrow 0} \frac{\ln(1-4x)}{x} = \lim_{t \rightarrow 0} \frac{\ln(1+t)}{-\frac{t}{4}} = \lim_{t \rightarrow 0} -4 \frac{\ln(1+t)}{t} = -4 \cdot 1 = -4$$

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$$\lim_{x \rightarrow 0} \frac{e^{-2x} - 1}{x} = \frac{0}{0} \quad \mathbf{F. I.}$$

ponendo $t = -2x$, $t \rightarrow 0$ per $x \rightarrow 0$ e il limite diventa:

$$\lim_{x \rightarrow 0} \frac{e^{-2x} - 1}{x} = \lim_{t \rightarrow 0} \frac{e^t - 1}{-\frac{t}{2}} = \lim_{t \rightarrow 0} -2 \frac{e^t - 1}{t} = -2 \cdot 1 = -2$$