FORMULARIO: goniometria

DEFINIZIONI

» FUNZIONI GONIOMETRICHE

$sen\ \alpha = \frac{PQ}{OP}$,

$\cos\alpha = \frac{OQ}{OP}$

$tg\ \alpha = \frac{sen\ \alpha}{\cos\ \alpha} = \frac{PQ}{OQ}$

» FUNZIONI RECIPROCHE

$cosec\ \alpha = \frac{1}{sen\ \alpha}$,

$\sec\alpha = \frac{1}{\cos\alpha}$,

$ctg\ \alpha = \frac{\cos\ \alpha}{sen\ \alpha} = \frac{OQ}{PQ}$

» LIMITAZIONI

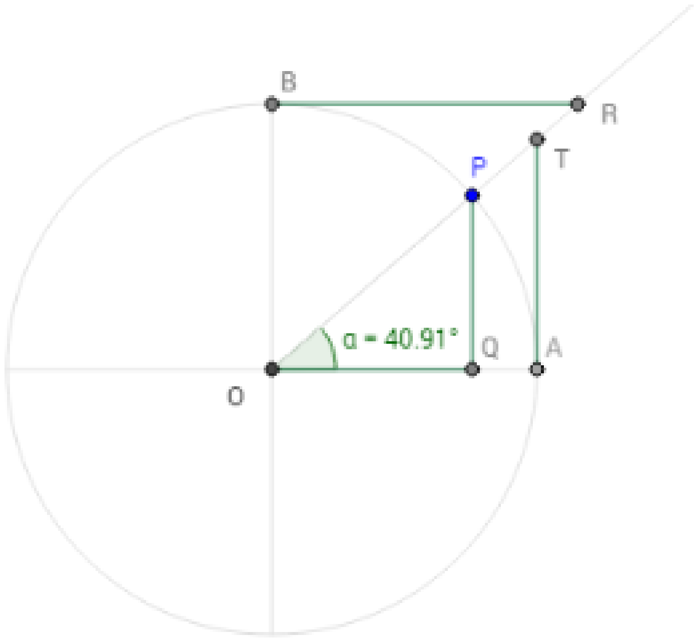
$-1 \leq sen\,\alpha \leq 1$,

$-1 \leq \cos\alpha \leq 1$

» RELAZIONE FONDAMENTALE DELLA GONIOMETRIA

$$sen^2\alpha + \cos^2\alpha = 1$$

» SIGNIFICATO GEOMETRICO DELLE FUNZIONI GONIOMETRICHE



$sen\alpha = \frac{PQ}{OP} = 0.65$ $\cos\alpha = \frac{OQ}{OP} = 0.76$

$tg\alpha = \frac{PQ}{OQ} = AT = 0.87$ $ctg\alpha = \frac{OQ}{PQ} = BR = 1.15$

Muovi il punto P per vedere come variano le lunghezze dei segmenti orientati.

$sen\ \alpha = PQ$ $\cos\ \alpha = OQ$ $tg\ \alpha = AT$ $ctg\ \alpha = BR$

ESPRESSIONE DI TUTTE LE FUNZIONI GONIOMETRICHE DI UN ANGOLO ORIENTATO MEDIANTE UNA SOLA DI ESSE

NOTO	$sen\,\alpha$	$\cos\alpha$	$tg\,\alpha$	$ctg\,\alpha$
$sen\,\alpha$	$sen\,\alpha$	$\pm\sqrt{1-\cos^2\alpha}$	$\frac{tg\,\alpha}{\pm\sqrt{1+tg^2\alpha}}$	$\frac{1}{\pm\sqrt{1+ctg^2\alpha}}$
$\cos\alpha$	$\pm\sqrt{1-sen^2\alpha}$	$\cos\alpha$	$\frac{1}{\pm\sqrt{1+tg^2\alpha}}$	$\frac{ctg\,\alpha}{\pm\sqrt{1+ctg^2\alpha}}$
$tg\,\alpha$	$\frac{sen\,\alpha}{\pm\sqrt{1-sen^2\alpha}}$	$\frac{\pm\sqrt{1-\cos^2\alpha}}{\cos\alpha}$	$tg\,\alpha$	$\frac{1}{ctg\,\alpha}$
$ctg\,\alpha$	$\frac{\pm\sqrt{1-sen^2\alpha}}{sen\,\alpha}$	$\frac{\cos\alpha}{\pm\sqrt{1-\cos^2\alpha}}$	$\frac{1}{tg\,\alpha}$	$ctg\,\alpha$

ARCHI ASSOCIATI

ANGOLI COMPLEMENTARI	ANGOLI CHE DIFFERISCONO DI UN ANGOLO RETTO
$sen\left(\frac{\pi}{2}-\alpha\right)=\cos\alpha$ $\cos\left(\frac{\pi}{2}-\alpha\right)=sen\,\alpha$ $tg\left(\frac{\pi}{2}-\alpha\right)=ctg\,\alpha$ $ctg\left(\frac{\pi}{2}-\alpha\right)=tg\,\alpha$	$sen\left(\frac{\pi}{2}+\alpha\right)=\cos\alpha$ $\cos\left(\frac{\pi}{2}+\alpha\right)=-sen\,\alpha$ $tg\left(\frac{\pi}{2}+\alpha\right)=-ctg\,\alpha$ $ctg\left(\frac{\pi}{2}+\alpha\right)=-tg\,\alpha$
ANGOLI CHE HANNO PER SOMMA TRE ANGOLI RETTI	ANGOLI CHE DIFFERISCONO DI TRE ANGOLI RETTI
$sen\left(\frac{3\pi}{2}-\alpha\right)=-\cos\alpha$ $\cos\left(\frac{3\pi}{2}-\alpha\right)=-sen\,\alpha$ $tg\left(\frac{3\pi}{2}-\alpha\right)=ctg\,\alpha$ $ctg\left(\frac{3\pi}{2}-\alpha\right)=tg\,\alpha$	$sen\left(\frac{3\pi}{2}+\alpha\right)=-\cos\alpha$ $\cos\left(\frac{3\pi}{2}+\alpha\right)=sen\,\alpha$ $tg\left(\frac{3\pi}{2}+\alpha\right)=-ctg\,\alpha$ $ctg\left(\frac{3\pi}{2}+\alpha\right)=-tg\,\alpha$
ANGOLI CHE DIFFERISCONO DI UN ANGOLO PIATTO	ANGOLI SUPPLEMENTARI
$sen(\pi+\alpha)=-sen\,\alpha$ $\cos(\pi+\alpha)=-\cos\alpha$ $tg(\pi+\alpha)=tg\,\alpha$ $ctg(\pi+\alpha)=ctg\,\alpha$	$sen(\pi-\alpha)=sen\,\alpha$ $\cos(\pi-\alpha)=-\cos\alpha$ $tg(\pi-\alpha)=-tg\,\alpha$ $ctg(\pi-\alpha)=-ctg\,\alpha$
ANGOLI ESPLEMENTARI	ANGOLI OPPOSTI
$sen(2\pi-\alpha)=-sen\,\alpha$ $\cos(2\pi-\alpha)=\cos\alpha$ $tg(2\pi-\alpha)=-tg\,\alpha$ $ctg(2\pi-\alpha)=-ctg\,\alpha$	$sen(-\alpha)=-sen\,\alpha$ $\cos(-\alpha)=\cos\alpha$ $tg(-\alpha)=-tg\,\alpha$ $ctg(-\alpha)=-ctg\,\alpha$

FORMULE DI ADDIZIONE E SOTTRAZIONE

$$sen(\alpha\pm\beta)=sen\,\alpha\cos\beta\pm sen\,\beta\cos\alpha$$

$$\cos(\alpha\pm\beta)=\cos\alpha\cos\beta\mp sen\,\alpha\, sen\,\beta$$

$$tg(\alpha\pm\beta)=\frac{tg\,\alpha\pm tg\,\beta}{1\mp tg\,\alpha\, tg\,\beta}$$

FORMULE DI DUPLICAZIONE

$$sen\,2\alpha = 2\,sen\alpha\,\cos\alpha$$
$$\cos 2\alpha = \cos^2\alpha - sen^2\alpha = 1 - 2sen^2\alpha = 2\cos^2\alpha - 1$$
$$tg\,2\alpha = \frac{2tg\,\alpha}{1 - tg^2\alpha}$$

FORMULE DI BISEZIONE

$$sen\frac{\alpha}{2} = \pm\sqrt{\frac{1-\cos\alpha}{2}}, \quad \cos\frac{\alpha}{2} = \pm\sqrt{\frac{1+\cos\alpha}{2}}, \quad tg\frac{\alpha}{2} = \pm\sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}}$$

FORMULE PARAMETRICHE

$$sen\,\alpha = \frac{2tg\frac{\alpha}{2}}{1+tg^2\frac{\alpha}{2}}, \quad \cos\alpha = \frac{1-tg^2\frac{\alpha}{2}}{1+tg^2\frac{\alpha}{2}}, \quad tg\alpha = \frac{2tg\frac{\alpha}{2}}{1-tg^2\frac{\alpha}{2}} \qquad \alpha \neq \pi + 2k\pi$$

FORMULE DI WERNER

$$sen\,\alpha\,sen\,\beta = \frac{1}{2}\left[\cos(\alpha-\beta) - \cos(\alpha+\beta)\right]$$
$$\cos\alpha\cos\beta = \frac{1}{2}\left[\cos(\alpha+\beta) + \cos(\alpha-\beta)\right]$$
$$sen\,\alpha\cos\beta = \frac{1}{2}\left[sen(\alpha+\beta) + sen(\alpha-\beta)\right]$$

FORMULE DI PROSTAFERESI

$$sen\,\alpha + sen\,\beta = 2\,sen\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$$
$$sen\,\alpha - sen\,\beta = 2\,sen\frac{\alpha-\beta}{2}\cos\frac{\alpha+\beta}{2}$$
$$\cos\alpha + \cos\beta = 2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$$
$$\cos\alpha - \cos\beta = -2\,sen\frac{\alpha+\beta}{2}\,sen\frac{\alpha-\beta}{2}$$

FORMULE DI BRIGGS

$$sen\frac{\alpha}{2} = \sqrt{\frac{(p-b)\cdot(p-c)}{bc}}, \quad sen\frac{\beta}{2} = \sqrt{\frac{(p-a)\cdot(p-c)}{ac}}, \quad sen\frac{\gamma}{2} = \sqrt{\frac{(p-a)\cdot(p-b)}{ab}}$$
$$\cos\frac{\alpha}{2} = \sqrt{\frac{p\cdot(p-a)}{bc}}, \quad \cos\frac{\beta}{2} = \sqrt{\frac{p\cdot(p-b)}{bc}}, \quad \cos\frac{\gamma}{2} = \sqrt{\frac{p\cdot(p-c)}{bc}}$$

$$tg\frac{\alpha}{2}=\sqrt{\frac{(p-b)\cdot(p-c)}{p\cdot(p-a)}},\quad tg\frac{\beta}{2}=\sqrt{\frac{(p-a)\cdot(p-c)}{p\cdot(p-b)}},\quad tg\frac{\gamma}{2}=\sqrt{\frac{(p-a)\cdot(p-b)}{p\cdot(p-c)}}$$

$$ctg\frac{\alpha}{2}=\sqrt{\frac{p\cdot(p-a)}{(p-b)\cdot(p-c)}},\quad ctg\frac{\beta}{2}=\sqrt{\frac{p\cdot(p-b)}{(p-a)\cdot(p-c)}},\quad ctg\frac{\gamma}{2}=\sqrt{\frac{p\cdot(p-c)}{(p-a)\cdot(p-b)}}$$

FORMULE DI NEPERO

$$\frac{a+b}{a-b}=\frac{tg\frac{\alpha+\beta}{2}}{tg\frac{\alpha-\beta}{2}},\quad \frac{a+b}{a-b}=\frac{ctg\frac{\gamma}{2}}{tg\frac{\alpha-\beta}{2}}$$

FUNZIONI GONIOMETRICHE DI ANGOLI PARTICOLARI **[Tabella completa pdf]**

Gradi	Radianti	*sen*	*cos*	*tg*	*ctg*
0°	0	0	1	0	non esiste
30°	$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$	$\sqrt{3}$
45°	$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1	1
60°	$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{\sqrt{3}}{3}$
90°	$\frac{\pi}{2}$	1	0	non esiste	0
180°	π	0	-1	0	non esiste
270°	$\frac{3\pi}{2}$	-1	0	non esiste	0
360°	2π	0	1	0	non esiste